Optimal Design of Solar PV Farms With Storage

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***Abstract*—We consider the problem of allocating a capital budget to solar panels and storage to maximize the expected revenue in the context of a large-scale solar farm participating in an energy market. This problem is complex due to many factors. To begin with, solar energy production is stochastic, with a high peak-to-average ratio, thus the access link is typically provisioned at less than peak capacity, leading to the potential waste of energy due to curtailment. The use of storage prevents power curtailment, but the allocation of capital to storage reduces the amount of energy produced. Moreover, energy storage devices are imperfect. A solar farm owner is thus faced with two problems: 1) deciding the level of power commitment and 2) the operation of storage to meet this commitment. We formulate two problems corresponding to two different power commitment approaches, an optimal one and a practical one, and show that the two problems are convex, allowing efficient solutions. Numerical examples show that our practical power commitment approach is close to optimal and also provide several other engineering insights.**

***Index Terms*—Batteries, budget splitting, solar PV panels.**

I. Introduction

GRID-SCALE solar farms are being rapidly deployed around the world today, with India and China alone planning to add 100 GW of solar power each in the next 5–7 years. Nearly all solar farms being deployed today lack storage: solar production is either directly absorbed into the grid or *curtailed*, where the curtailment is either due to inadequate access line capacity or to maintain grid stability. Today, solar farm operators are typically paid for solar energy production whether or not it is curtailed, so they are indifferent to curtailment. However, curtailment comes at a significant financial cost to grid operators, who need to pay for energy although, it cannot be used to meet loads.

As solar generation becomes an increasingly larger fraction of the overall energy supply, the cost of curtailment is likely to become unsustainable. We anticipate that this will force solar farm operators to become more like traditional generators, in that they will need to commit to a certain constant power level for a certain duration (we call this a *market time slot*) a day or an hour in advance, receiving revenue for this committed power, and that they will need to pay a penalty if this commitment is not met. To avoid penalties or, more generally, to maximize their revenue, such a change in solar farm operation will make it necessary for farm owners to invest in storage that smooths out power fluctuations. In this paper, we study the optimal allocation of a fixed budget to solar panels and storage in this future price regime.

More specifically, in this regime, the amount of storage that needs to be purchased by a solar farm operator is influenced by six distinct, inter-related factors.

1) The *capacity of the access link* that connects the farm to the rest of the grid: the smaller this capacity, the higher the possibility of curtailment, and the greater the need for storage.
2) The *power level committed* to by the farm owner in each market time slot: the higher the commitment level, the higher the probability of not meeting this commitment, and hence the greater the need for storage.
3) The *penalty*: the higher the penalty, the greater the motivation to either choose a lower commitment level or to store energy to prevent a future shortfall.
4) The degree of *fluctuation in the purchase price in different market time slots*: the greater the fluctuation, the larger the benefit from storing solar production for future sale at a higher price.
5) The degree of *variation in solar energy* during a single market time slot: the greater this variation, the greater the need for storage.
6) The *solar generation prediction accuracy*: the lower the prediction accuracy, the greater the need for storage to mitigate against prediction errors.

Due to the complex relationship between these factors, the optimal allocation of resources to solar panels and storage is a challenging problem. In this paper, we determine, at design time, the optimal budget allocation such that the system, when commissioned, would maximize its *anticipated* revenue in the day-ahead market.

What prices would solar generation receive in future markets? It is evident that the current pricing scheme (which is a fixed feed-in tariff with no penalties) must be changed to reduce the burden on the grid to smooth-out renewable generation. It is, however, not clear how the pricing schemes for solar will play out. To simplify the problem, in this paper, we assume the simplest possible scenario, which is to assign constant reward and penalty prices. We also assume perfect prediction of solar power. We defer the more general setting, with variable power prices and imperfect prediction, to future work.

The prior work closest to ours is a study of the impact of access link capacity and purchase price fluctuation on the amount of storage that needs to be purchased by solar farm

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owners in Puglia and Badem-Wuttemberg [1]. Our problem formulation is different in that we study the effect of market pricing schemes (penalty price, the length of a market time slot, and power-level commitment) as well as access link capacity on the optimal budget split. Our key contributions include:

1) modeling a complex system that includes stochastic solar power production, realistic storage systems, and both optimal and practical power commitment approaches, formulating the corresponding problems to obtain an optimal budget split between solar generation and storage;
2) showing that the two problems are convex and that the optimal storage charge and discharge operation can be described with simple rules;
3) engineering insights that shed new light on the role of the length of the market time slot, penalty, and access link capacity on the optimal budget split.

This paper is organized as follows. We discuss the existing work on solar farm designs in Section II. We present the system model and notation in Section III. In Section IV, we formulate the problems and prove two useful lemmas. We present numerical examples in Section V and conclude the paper in Section VI.

II. Related Work

There is extensive work on sizing or analyzing the performance of storage-photovoltaic (PV) systems. Prior work can be categorized into two main classes: *stand-alone* problems and *grid-connected* problems (see [2]–[4] for an extensive review of existing related work).

Stand-alone problems are those in which the system can only rely on solar power and storage to meet the demand power. Several papers studied the optimal sizing and cost analysis of stand-alone PV systems [2], [5]–[7]. The objective in stand-alone systems is to minimize the cost of the battery-PV system, while still meeting the power demand with a target loss of load probability. Cost minimization is either in terms of minimizing the initial capital cost of the system [8], [9] or the annualized cost of the system accounting for different lifetime of batteries and PV panels [10], [11]. Annualized cost minimization is further extended to a general target output power in [12].

Grid-connected scenarios themselves are divided into two classes: residential/commercial installations and solar PV farms. Residential/commercial installations of PV systems have the option to serve demand from PV panels, storage, or the grid. Typically, the price of buying/selling electricity to the grid is a function of the time of the day and season. Such installations mostly aim at selling their excess power to the grid and buying their power shortage from the grid. These options create many challenging problems with different objective functions. Barra *et al.* [13] optimally size PV panels and storage such that a minimum target fraction of the total demand is guaranteed to be met by the battery-PV system and the cost of energy is minimized. Azzopardi and Mutale minimize the annual net cost, using a case study of a residential installation where energy can be stored, used, or sold [14]. They consider fluctuations in time of use pricing and use mixed integer programming to find the optimal size of each system component. Using a similar

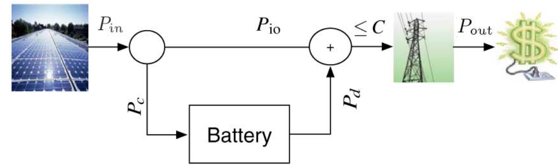

Fig. 1. System model.

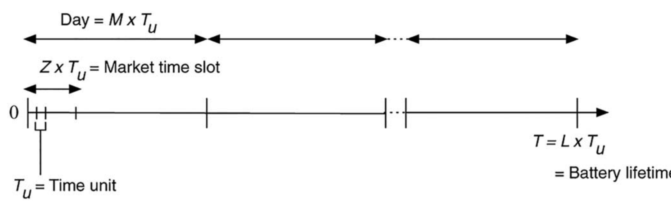

Fig. 2. Illustrating the time unit definitions.

system model, Ru *et al.* [15] provide an optimization problem to determine the critical size of the battery after which an increase in size gives no performance benefit. Other work maximizes the benefit minus cost of a grid-connected solar PV panel with no storage [16], [17]. The joint optimal technology selection and operation to minimize electricity bills of a commercial building is considered in [18] and [19]. Our work is targeted at PV farms that have no intrinsic electricity demand of their own, and thus these results do not carry over.

Prior work on solar farm sizing does not focus on the allocation of a fixed budget to either solar production or storage. Typically, the level of investment in the solar panels is given, and the question is whether or not to invest in storage [1]. Moreover, in most prior work, there is no penalty for not meeting demand [1]–[4]. Finally, prior work is mostly focused on meeting a given demand with some target allowable uncertainty [3], [12]. The target-committed power level is a free variable in our optimization problem. The joint optimization of the budget split and the target-committed power level makes the problem challenging. These differences result in a problem structure in prior work that differs substantially from ours; our formulation allows us to derive some engineering insights on market time-slot duration and power commitment level that cannot be obtained from prior work.

III. System Model

Fig. 1 illustrates our system, consisting of solar PV panels and a battery. We assume a discrete-time model, where time is slotted; $k = 0, T_u, 2T_u, \ldots$, with T_u being the time unit. We assume that the changes can happen only at the beginning of each time slot and all inputs and parameters are constant during a time slot. We assume that $k = 0$ is the time the PV farm system in Fig. 1 is created. The available power from solar PV panels at any time k is $P_{\rm in}(k)$. The actual output power from the solar PV farm $P_{\rm out}(k)$ is transmitted over an access line of capacity of C power units to the grid. We assume a day-ahead market and a market time slot of duration ZT_u (see Fig. 2). The farm owner commits to delivering a constant power during each market time slot. The target-committed output power during market time slot j in day d is denoted by $\Pi(j, d)$. Note

Name	*Description (units)*
C	Access capacity (MW)
K	Total available budget ($)
K_{pv}	Total investment on PV panels
K_B	Total investment on battery
c	Revenue for each energy unit ($/MWh)
p	Penalty for each unmet energy unit ($/MWh)
k	Time slot index, each slot is of length T_u
T_u	The size of a time slot (h)
M	Number of time slots in a day
$T_{j,d}$	The set of time slots in day d and market time slot j
Z	The duration of a market time slot in number of time units T_u
$Rev(N)$	Total revenue in the first N time slots ($)
Rev_{tot}	The overall revenue of the system
α_{pv}	The efficiency of solar PV panels
A	Total surface area of solar PV panels (m^2)
$P_{max}(A)$	The maximum possible power production from PV panels of size A (MW)
u	Price per unit of 1MW PV panel ($/MWh)
r_L	Discounted value of PV panels after battery lifetime
B	Battery size (MWh)
v	Price per unit of battery ($/MWh)
T	Lifetime of the battery (h)
L	Lifetime of the battery (# of time slots)
$\alpha_c(\alpha_d)$	Battery charging (discharging) rate limit (h^{-1})
$\eta_c(\eta_d)$	Battery charging (discharging) efficiency
γ	Battery self-discharge rate
a_1	Minimum chargeable fraction level of the battery
a_2	Maximum chargeable fraction level of the battery
$i(k)$	Solar irradiance in time slot k (MW/m^2)
$i_n(k)$	Normalized solar irradiance in time slot k (MW/m^2)
$P_{in}(k)$	The available solar power in time slot k (MW)
$P_{out}(k)$	The output power from PV farm in time slot k (MW)
$P_{io}(k)$	The output power from input in time slot k (MW)
$P_s(k)$	The committed output power in time slot k (MW)
$P_c(k)$	The storage charging power in time slot k (MW)
$P_d(k)$	The output power from storage in time slot k (MW)
$\Pi(j,d)$	The committed output power in market time slot $T_{j,d}$ (MW)

that this commitment cannot exceed the access line capacity; thus

$$0 \leq \Pi(j,d) \leq C \quad \forall j,\, \forall d. \tag{1}$$

In our problem formulation, $\Pi(j,d)$ for each market time slot j in day d is a control variable, so that this choice of power commitment is a decision variable, chosen to maximize expected revenue.

We denote by $P_{\mathrm{io}}(k)$ and $P_d(k)$, respectively, the portions of the output power that come directly from the input solar power and from the battery. Define $T_{j,d}$ to be the time interval corresponding to the time slots in day d and market time slot j in that day, i.e.,

$$T_{j,d} = \{k | k \in [(d-1)M + (j-1)Z + 1, (d-1)M + jZ]\} \tag{2}$$

where M is the number of time slots in a day. Thus, we can write

$$P_{\mathrm{out}}(k) = P_{io}(k) + P_d(k) \leq \Pi(j,d) \quad \forall k \in T_{j,d}, \forall j, \forall d \tag{3}$$
$$P_s(k) = \Pi(j,d) \quad \forall k \in T_{j,d}, \forall j, \forall d \tag{4}$$

where the first inequality implies that the entire system might fail to provide the target output power at certain times and it is never larger than the target output power.

Given our notation, the system model in Fig. 1 has the following constraints ($\forall k$):

$$0 \leq P_d(k) + P_{\mathrm{io}}(k) \leq P_s(k) \tag{5}$$
$$0 \leq P_c(k) + P_{\mathrm{io}}(k) \leq P_{\mathrm{in}}(k) \tag{6}$$
$$0 \leq P_c(k), P_{\mathrm{io}}(k), P_d(k). \tag{7}$$

Besides these constraints, we model battery imperfections as follows. The charging (discharging) power must not exceed $\alpha_c B$ ($\alpha_d B$) at any time.[1] The battery loses a fraction of $1 - \eta_c$ ($1 - \eta_d$) when charging (discharging), because of battery charging (discharging), inefficiency due to energy conversion losses.[2] To achieve a reasonable battery lifetime, the battery state of charge should not violate a minimum of $a_1 B$ and a maximum of $a_2 B$. Finally, the stored energy is reduced by a fraction $1 - \gamma \leq 1$ after each time unit, due to self-discharge. In summary, if $b(k)$ is the state of charge evolution in time slot k, then we have

$$b(0) = a_1 B \tag{8}$$
$$b(k) = (1-\gamma)b(k-1) + \eta_c P_c(k) T_u - P_d(k) T_u / \eta_d \tag{9}$$
$$a_1 B \leq b(k) \leq a_2 B \tag{10}$$
$$0 \leq P_d(k) \leq \alpha_d B \tag{11}$$
$$0 \leq P_c(k) \leq \alpha_c B \tag{12}$$
$$P_c(k) \times P_d(k) = 0 \tag{13}$$

where B is the battery size and the last equality ensures that we cannot charge and discharge the battery simultaneously.

The output power $P_{\mathrm{out}}(k)$ is transmitted via the access line to be sold in an electricity market and by design will not be larger than $P_s(k)$. We believe that the day-ahead electricity market is likely to use one of the two following policies to pay/charge solar PV owners. In the first policy, the supplier earns $\$c_1$ for each energy unit it produces, and pays a penalty of $\$p_1$ for each energy unit it falls short during the day of operation. Thus, the total revenue in the first N time slots in this policy is given by

$$\mathrm{Rev}_1(N) = \sum_{k=1}^{N} \left(c_1 P_{\mathrm{out}}(k) - p_1(P_s(k) - P_{\mathrm{out}}(k))\right) T_u. \tag{14}$$

In the second policy, everything is the same as before, except that the supplier is rewarded for its *target* output power $P_s(k)$ (which is constant during each market time slot $T_{j,d}$) rather than its actual production P_{out}. In this case, the revenue in the first N time slots is

$$\mathrm{Rev}_2(N) = \sum_{k=1}^{N} \left(c_2 P_s(k) - p_2(P_s(k) - P_{\mathrm{out}}(k))\right) T_u \tag{15}$$

[1] Here, we assume a homogenous battery sizing, in which increasing the size of the battery is done by adding identical cells of the same technology from the same manufacturer.

[2] We can also incorporate the inefficiency of the converters by treating η_c (η_d) to be the product of the charging (discharging) inefficiencies of the battery and the converters.

where $\$c_2$ and $\$p_2$ are, respectively, the per-energy unit reward and penalty. However, the revenue formulations in these two policies from (14) and (15) are equivalent if we redefine the penalty price by setting $c_2 = c_1$ and $p_2 = c_1 + p_1$, keeping in mind that $p_2 \geq c_1$. Note that both (14) and (15) are written in constant dollars, i.e., they are already built in a nominal rate of inflation. Thus, when we compute the net revenue, the penalty and reward values are assumed to scale at the same rate. Thus, the formulations are independent of the *time-value of money* and *discount rates*.

Following existing work (e.g., [20] and [21]), we choose (15) to be the objective function for our problem. We simplify notation by using c and p instead of c_2 and p_2. Combining (4) and (15) with some manipulations yields

$$\mathrm{Rev}(N) = \sum_{k=1}^{N} \left((c-p)P_s(k) + p\left(P_d(k) + P_{\mathrm{io}}(k)\right)\right) T_u. \tag{16}$$

Given a total budget of $\$K$, our goal is to optimally size a solar PV farm (illustrated in Fig. 1) with the maximum revenue (16) over its lifetime. The budget can be used to buy either solar PV panels or batteries. In Section IV, we formulate and discuss this problem in greater detail.

IV. Problem Definition

In this section, we discuss the problem formulation.

A. Optimal Budget Split

Our first goal is to estimate, from a solar irradiance trace, the cost of the solar panels needed to produce a certain peak power. Let us denote K_{pv} to be the total budget invested for PV panels. Denote by $i(k)$ the available solar power at any time k per unit area at the given location. The available power from solar PV panels P_{in} is given by

$$P_{\mathrm{in}}(k) = \alpha_{\mathrm{pv}} A i(k) \tag{17}$$

where α_{pv} is the efficiency of the solar PV panels and A is the total surface area. We define $P_{\max}(A)$ to be the peak power produced by a PV farm with total surface area A. Then, from (17), we have

$$P_{\max}(A) = \max_{k'}(i(k'))\alpha_{\mathrm{pv}} A. \tag{18}$$

Given a cost per unit peak power u, a solar PV farm with total surface area A costs $P_{\max}(A)u$. Thus, for a given K_{pv}, the area of the PV panels affordable is given by

$$A = \frac{K_{\mathrm{pv}}}{u \max_{k'}(i(k'))\alpha_{\mathrm{pv}}}. \tag{19}$$

Inserting this value in (17) yields

$$\begin{aligned} P_{\mathrm{in}}(k) &= \frac{K_{\mathrm{pv}}}{u \max_{k'}(i(k'))} i(k) \\ &= \frac{K_{\mathrm{pv}}}{u} i_n(k) \end{aligned} \tag{20}$$

where $i_n(k)$ is the normalized irradiance given by

$$i_n(k) := \frac{i(k)}{\max_{k'}(i(k'))}. \tag{21}$$

Given a K_{pv}, we can compute the corresponding PV panel size and P_{in}, respectively, from (19) and (20).

Define K_B to be the total budget invested for batteries. The overall budget is either invested in battery purchase or in buying PV panels. Thus,

$$K_{\mathrm{pv}} + K_B = K. \tag{22}$$

Thus, the size of battery is given by

$$B = \frac{K_B}{v} \tag{23}$$

where v is the price per unit of storage.

B. PV Panel Discounted Price

The lifetime of a PV panel is typically much longer than the lifetime of a battery. Therefore, at the end of the lifetime of the battery, the PV panels are still likely to be functional. This must be taken into account in the revenue formulation. We assume that the asset value of each dollar invested on PV panels at the end of the lifetime of the battery is reduced by a discounted factor $r_L \leq 1$, where L is the lifetime of the battery in time units. The solar asset decay r_L occurs due to two factors: the decreasing price of PV panels mostly due to improvement in the technology and the depreciation of the original asset. We assume a 12% annual decay in solar asset worth and an additional 5% annual decrease in the original worth of the panels, which follows from their 20-year lifetime. Thus,

$$r_L = (0.88)^L \left(1 - \frac{L}{20}\right). \tag{24}$$

By the end of the lifetime of the battery, the asset value of the PV panels is $\$r_L K_{pv}$ and it must be included in the total revenue

$$\begin{aligned} \mathrm{Rev}_{\mathrm{tot}} &= \mathrm{Rev}(L) + r_L K_{\mathrm{pv}} \\ &= \sum_{k=1}^{L}((c-p)P_s(k) + p\left(P_d(k) + P_{\mathrm{io}}(k)\right)) T_u + r_L K_{\mathrm{pv}} \end{aligned} \tag{25}$$

where in the second line, we inserted the revenue during the lifetime of the battery from (16).

C. Choice of Power Commitment $\Pi(j, d)$

Solar power generation is diurnal. A solar farm owner should commit to a target power level $\Pi(j, d)$ that roughly follows the diurnal variations in solar power, i.e., the average expected solar power production during each market time slot[3] [22]. Thus, a

[3]Note that this is easier than to forecast the exact time series of future irradiance values.

possible candidate for power commitment is what we call the *average commitment*, i.e.,

$$\Pi(j,d) = \min(C, \overline{P}_{\text{in}}(j,d)) \quad \forall j, \forall d \tag{26}$$

where $\overline{P}_{\text{in}}(j,d)$ is the average solar power over market time slot $T_{j,d}$, given by

$$\overline{P}_{\text{in}}(j,d) := \frac{\sum_{k' \in T_{j,d}} P_{\text{in}}(k')}{Z}. \tag{27}$$

However, there might be reasons why committing to the average over a period is not the right thing to do. For example, if the battery is empty at the beginning of the period, the system would not be able to cope with a possible shortfall during the first part of the time period. If the battery is full, it might be possible to commit more than the average. Hence, in the following, we will evaluate the average commitment by comparing it to an upper bound on the revenue. Precisely, we will consider the two following problems.

1) P1 (optimal commitment): In this formulation, the power commitment $\Pi(j,d)$ is a free parameter. Therefore, the solver finds the optimum choice of $\Pi(j,d)$ for the given inputs. We use this result as a benchmark, i.e., to obtain an upper bound on the revenue. This problem, however, will not give us any insights into how to perform power commitment.
2) P2 (average commitment): In this formulation, we assume that the power commitment is given by (26).

We now formulate these two problems.

D. Formulating P1

Using the above definitions and notation and given K, C, L, T_u, c, p, $i(k)$, u, v, and the battery imperfections (α_c, α_d, η_c, η_d, a_1, a_2, and γ), we can write the solar PV farm design optimization problem $P1$ as follows:

$$P1: \max_{K_{\text{pv}}, P_d(k), P_{\text{io}}(k), P_c(k), \Pi(j,d)} \sum_{k=1}^{L} ((c-p)P_s(k) + p(P_d(k)+P_{\text{io}}(k)))\, T_u + r_L K_{\text{pv}} \tag{28}$$

s.t.

$$\text{Ct1}: K_{\text{pv}} + K_B = K$$
$$\text{Ct2}: B = \frac{K_B}{\nu}$$
$$\text{Ct3}: b(0) = a_1 B$$
$$\text{Ct4}: b(k) = (1-\gamma)b(k-1) + \eta_c P_c(k) T_u - P_d(k) T_u/\eta_d \quad \forall k$$
$$\text{Ct5}: a_1 B \le b(k) \le a_2 B \quad \forall k$$
$$\text{Ct6}: 0 \le P_d(k) + P_{\text{io}}(k) \le P_s(k) \quad \forall k$$
$$\text{Ct7}: 0 \le P_c(k) + P_{\text{io}}(k) \le P_{\text{in}}(k) \quad \forall k$$
$$\text{Ct8}: 0 \le P_c(k), P_{\text{io}}(k), P_d(k) \quad \forall k$$
$$\text{Ct9}: 0 \le P_d(k) \le \alpha_d B \quad \forall k$$
$$\text{Ct10}: 0 \le P_c(k) \le \alpha_c B \quad \forall k$$
$$\text{Ct11}: P_{\text{in}}(k) = \frac{K_{\text{pv}}}{u} i_n(k) \quad \forall k$$
$$\text{Ct12}: 0 \le \Pi(j,d) \le C \quad \forall j, \forall d$$
$$\text{Ct13}: P_s(k) = \Pi(j,d) \quad \forall k \in T_{j,d}, \forall j, \forall d$$

$$\text{Ct14}: P_c(k) \times P_d(k) = 0 \quad \forall k.$$

The above optimization problem is nonlinear, because of the last constraint (Ct14). We, however, prove in the following lemma that this inequality can be safely removed, making the problem a linear programming (LP).

Lemma 1: The problem P1 without the last constraint, i.e., Ct14, always has a solution for which Ct14 is true and this solution would yield the same K_{pv}.

Note: From this point on, to simplify notation, whenever we refer to P1 we refer to the problem without Ct14, using the above lemma. This is why we separated Ct14 from the other constraints by a solid horizontal line.

Proof: Suppose that there exists a solution to P1 for which there exists at least one k such that $P_c^*(k) \times P_d^*(k) \neq 0$. Let us denote by $P_c^*(k)$, $P_d^*(k)$, and $P_{\text{io}}^*(k)$ the optimal charging, optimal discharging, and optimal feedforward power in time slot k. Let us define the following:

$$P_c'(k) = [P_c^*(k) - P_d^*(k)/(\eta_c \eta_d)]_+ \tag{29}$$
$$P_d'(k) = [P_d^*(k) - \eta_c \eta_d P_c^*(k)]_+ \tag{30}$$
$$P_{\text{io}}'(k) = \begin{cases} P_{\text{io}}^*(k) + P_d^*(k), & \text{if } \eta_c P_c^*(k) \ge P_d^*(k)/\eta_d \\ P_{\text{io}}^*(k) + \eta_c \eta_d P_c^*(k), & \text{if } \eta_c P_c^*(k) < P_d^*(k)/\eta_d \end{cases} \tag{31}$$

where $[y]_+ = \max(0, y)$ for any y. Then, it can be easily verified that replacing $P_c^*(k)$, $P_d^*(k)$, and $P_{\text{io}}^*(k)$, respectively, with $P_c'(k)$, $P_d'(k)$, and $P_{\text{io}}'(k)$ does not change neither the optimal revenue nor the optimal K_{pv} and satisfies all constraints including Ct14. ■

The physical interpretation of the proof is that we define $P_c'(k)$, $P_d'(k)$, and $P_{\text{io}}'(k)$ in a way to represent the net power flow to each element. For example, when we are simultaneously charging and discharging the battery, the overall net power flow to the battery is either positive or negative, and hence only one of $P_c'(k)$ or $P_d'(k)$ can be nonzero.

To recap, Ct14 guarantees that simultaneous charging and discharging cannot happen. With the above lemma, we can relax Ct14, and convert P1 to an LP.

E. Formulating P2

The power commitment $\Pi(i,j)$ in P2 is not a free parameter and is set to its value in (26). This is

$$P2: \max_{K_{\text{pv}}, P_d(k) P_{\text{io}}(k), P_c(k)} \sum_{k=1}^{L} ((c-p)P_s(k) + p(P_d(k) + P_{\text{io}}(k)))\, T_u + r_L K_{\text{pv}} \tag{32}$$

s.t.

Ct1 to $Ct12$

Ct15 : Eq. (26)

which is a nonlinear optimization problem, because of the constraint Ct15. This nonlinear optimization problem can be easily turned to an integer LP (ILP) by defining slack variables. Therefore, this is a convex optimization problem. This convex problem can be significantly simplified using the following lemma, which provides closed-form formulations for the optimal values of charging, discharging, and feedforward power.

Lemma 2 (optimal control strategy): The optimal values of P_c, P_d, and $P_{\rm io}$ in optimization problems P1 and P2 can be expressed in the following closed forms:

$$P_{\rm io}(k) = \min(P_s(k), P_{\rm in}(k)) \tag{33}$$

$$P_c(k) = \min([P_{\rm in}(k) - P_s(k)]_+, \alpha_c B, (a_1 B - (1-\gamma)b(k-1))/(\eta_c T_u)) \tag{34}$$

$$P_d(k) = \min([P_s(k) - P_{\rm in}(k)]_+, \alpha_d B, (1-\gamma)b(k-1)\eta_d/T_u). \tag{35}$$

See [23] for the proof.

Sketch of Proof: We first show that we should always maximize $P_{\rm io}$ regardless of P_d and P_c. Accordingly, the optimal value of $P_{\rm io}$ can be proved to match (33). Given this value for $P_{\rm io}$ and using Lemma 1 together with (5) and (6), we can show that the optimal values of P_d and P_c, respectively, match the values in (34) and (35). □

Based on the above lemma, the optimal control strategy in our problems follows these straightforward static rules: The input power $P_{\rm in}(k)$ is primarily used to serve the target output power, delivering $\min(P_{\rm in}(k), P_s(k))$ to the output line. The leftover (if any) $[P_{\rm in}(k) - P_s(k)]_+$ is stored. The energy is stored in the battery with power $P_c(k)$ at any time k. If, at a given time k, the available solar power is insufficient [i.e., $P_{\rm in}(k) < P_s(k)$], the energy stored in the battery, if any, can be used to make up the difference.

Storing energy in the battery when we have a chance to sell it is always harmful because: 1) there is no gain in terms of revenue in postponing selling energy; 2) we might lose some revenue because the battery may become full; and 3) we might lose stored energy due to self-discharge or charging/discharging efficiency of the batteries or converters.

Lemma 2 provides the optimal operation of the battery. This removes three free parameters from P2, leaving $K_{\rm pv}$ as the only one. This (given the convexity of P2) allows a simple hill-climbing technique to be used to solve problem P2.

V. Numerical Examples

We use the two approaches to design a solar PV farm with storage at a given location characterized by its irradiance trace. We compute the optimal revenue and the corresponding budget split for both P1 (using CPLEX) and P2 (using the hill-climbing technique coded in C++). We assume that the reward and penalty prices are set to $c = \$291/\text{MWh}$ and $p = 2 * c$, unless otherwise stated. The price (including hardware and installation) and the lifetime of a PV panel are set to $u = 1.63\$/\text{W}$ and 20 years, which are the typical values in 2014 [28]. We assume that our total initial budget is enough to build a 1-MW solar farm with no storage; thus , $K = \$1\,630\,000$. We use the solar irradiance dataset [i.e., $i(k)$ in our notation] from the atmospheric radiation measurement website [29] from the C1 station in the Southern Great Plains permanent site with a 5-min time resolution. Unless otherwise stated, the market time slot is fixed to 1 h ($Z = 60$) and the access line capacity is set to $C = 1.5 * P_{\rm av}$, where $P_{\rm av}$ is the average available solar power between 8 A.M. and 8 P.M. if the entire budget is spent on solar panels (the corresponding value is $C = 0.54$ MW). Although our problem formulation can be applied to a large set of batteries, we will only show results for lithium-ion (Li-ion) and lead-acid (PbA) batteries. Their characteristics are given in Table I.

TABLE I
Battery Characteristics at Room Temperature and Averaged Over Its Lifetime [24]–[27]

	Li-ion	PbA
DoD	0.8	0.8
Round-trip efficiency ($\eta_c\eta_d$)	0.85	0.75
Charge time ($1/\alpha_c$) (h)	1	4
Discharge time ($1/\alpha_d$)	0.5	0.5
Self-discharge (γ)	≈ 0	≈ 0
Lifespan (L_B) (years)	5	4
Per-unit price (v) ($/KWh)	400	200

A. Role of Budget Allocation and Market Time Scale

The overall revenue is greatly affected by the budget split between the panels and the batteries. Fig. 3 shows how the revenue is influenced by varying the budget split. We have repeated this example for two types of batteries (Li-ion and PbA), two market time-slot sizes ($Z = 15, 60$), and using both P1 and P2. This figure shows that there is not a significant difference between P1 and P2, suggesting that an average power commitment is a good choice. The revenue presents a convex behavior with respect to the budget split as also suggested by the convexity of both P1 and P2. As we increase the investment on solar PV panels, we increase the power production (advantageous) as well as the fluctuations (disadvantageous). As this figure shows, the maximal revenue occurs at $K_{\rm pv}$ ∼96%–99% of the total budget for both Li-ion and PbA batteries and both market time scales. Comparing the revenue in the presence of batteries compared to the case without batteries ($K_{\rm pv} = 1$), Fig. 3 shows that we have a 7.1% improvement in revenue using Li-ion and a 4.1% improvement using PbA batteries for $Z = 60$. The figure also shows that decreasing the market time period helps.

Finally, we conducted a sensitivity analysis with respect to the solar power traces, by repeating the above experiment on 100 modified solar sample paths. We found that these results are insensitive to the choice of solar power traces.

B. Role of the Line Access Capacity (C)

Fig. 4 shows the optimal revenue with and without storage for different values of access line capacity C, as a ratio of $P_{\rm av}$ for Li-ion batteries. Clearly, the revenue increases with an increase in line capacity C due to a reduction in curtailment, reaching a saturation point when the access capacity is no longer a binding

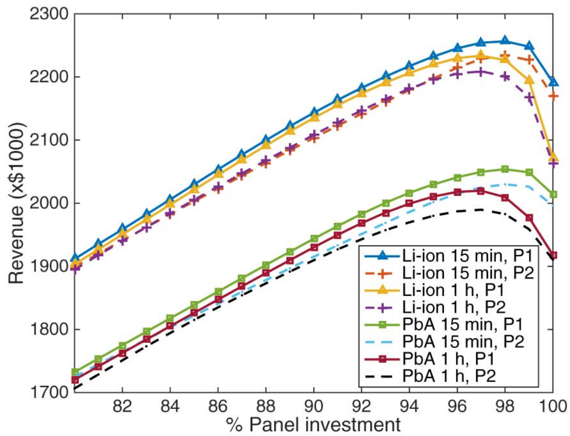

Fig. 3. Total revenue as a function of budget share (in percentage) for PV panels.

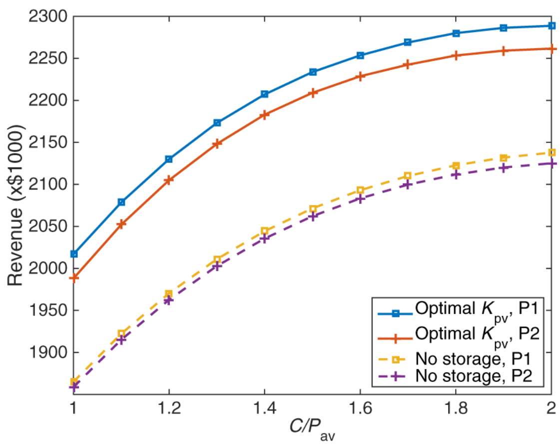

Fig. 4. Optimal revenue as a function of line access capacity C.

constraint. We also see that adding storage reduces the loss of revenue due to market penalties.

C. Sensitivity Analysis to Battery Lifetime

The battery lifetimes we use in our numerical examples are nominal averaged values. In practice, battery lifetime is highly dependent on how it is used, i.e., the detailed charge/discharge processes. We now discuss the sensitivity of the optimal budget split to the battery lifetime. We vary the battery lifetime from 1 to 20 years and record the corresponding optimal budget split for each point. As shown in Fig. 5, the optimal budget split is relatively insensitive to battery lifetime, changing by only a few percentage points. We see a slight shift toward a greater investment in batteries as the battery lifetime increases. The optimal budget split is even more insensitive to the battery lifetime if the battery lifetime is about (or larger than) its nominal lifetime as the vertical dashed line is close to the plateau of the curves in Fig. 5.

The monotonic decreasing trend of the optimal budget investment on solar PV panels reaches a saturation point as battery lifetime increases. This is due to the fact that the asset value of solar PV panels is almost negligible at 14% of its original value after about 10 years. Thus, the second term in (25),

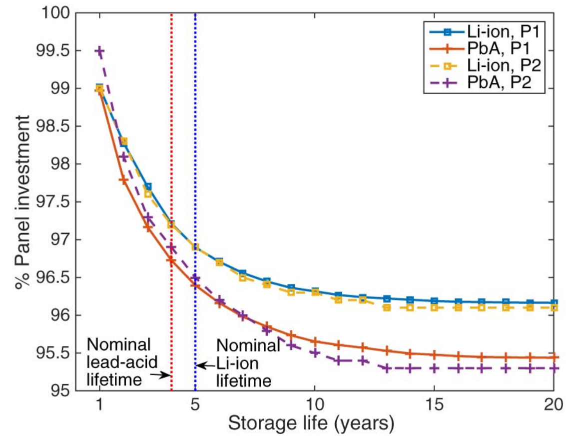

Fig. 5. Optimal budget split as a function of battery lifetime.

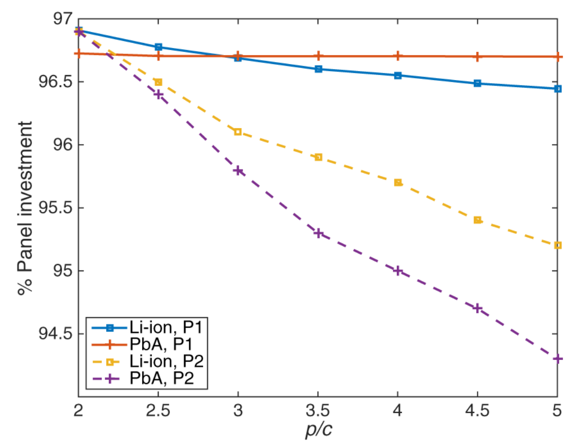

Fig. 6. Optimal budget split percentage for solar PV panels as a function of the penalty price.

which mainly reflects the impact of battery lifetime on the total revenue, becomes negligible.

D. Role of the Penalty Price Ratio (p/c)

Fig. 6 illustrates the impact of penalty prices on the optimal budget split. We take the ratio p/c as an independent variable by keeping c constant. The graph shows that the optimal budget split shifts in favor of investing in storage as the penalty price increases. This corroborates the fact that one of the primary roles of storage in this context is to smooth out the production and avoid paying the penalty; the larger the battery, the more likely we are able to smooth out the fluctuations in solar output with stored energy. For higher price ratios, the investment into batteries is slightly higher for PbA than for Li-ion. We observe that the difference between P1 and P2 increases as the penalty price increases but remains low. This is because the role of the choice of power commitment is more important for large penalty prices.

VI. Conclusion

Our work studies the one-shot optimal allocation of a capital budget to solar panels and storage to maximize anticipated revenue from a day-ahead or hour-ahead market over the lifetime of storage system (which is typically shorter than that of solar panels). Unlike prior work, we have carefully modeled several real-world constraints, yet have formulated two convex models that can be efficiently solved using hill-climbing or a solver. Numerical evaluation using real irradiation traces shows that it is typically optimal to invest 96%–99% of the initial investment on solar panels and the rest on storage. We find that this fraction depends on six inter-related factors, four of which we study in detail in this work. We find that the practical expedient of setting the power commitment level to the expected solar production during a market time slot results in nearly the same revenue as an optimal commitment level determined by a solver. Moreover, we find that as storage lifetimes increase, the optimal investment level in solar panels is about 96% and is nearly insensitive to lifetimes longer than about 10 years, which are already becoming possible with current technology.

The primary limitation of our work is that it assumes a very simple market structure, with constant revenue and penalty prices. We also assume a fixed line capacity C whose size cannot be increased through additional investments. We study only two types of batteries with homogenous cells. We hope to address these limitations in our future work. We also plan to work on improving our battery model and make it more realistic (e.g., accounting for state-of-health and the lifetime dependency on the charging/discharging operations).

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